Application of Fractal Geometry to 2D Fracture Networks in the Middle Atlas Aquifer (Morocco)

Mohamed Rouai
Geophysics Team, Earth Sciences Department, Meknes University
Meknes, Morocco
mrouai@fs-umi.ac.ma

SUMMARY

2D fracture network maps of Lias aquifer have been analyzed from their scaling properties. The fractal analysis of fracture intensity showed heterogeneous multifractal structure with characteristic generalized dimensions. Distribution of fracture lengths exhibits power-law behaviour with specific exponent. Scaling laws serve to make extrapolations, and to study the fracture connectivity related to scale, which are of great interest in decision-making to optimize ground-water prospecting.

KEYWORDS: *Fracture network, connectivity, fractals, multifractals, Morocco.*

INTRODUCTION

Quantitative understanding of spatial clustering and scaling behaviour of fracture patterns is fundamental to study fractured reservoir hydrology and the mechanical properties of discontinuous rocks.

In this work, we analyze the fractal (Mandelbrot, 1982; Turcotte, 1997) and multifractal properties of fracture network in the Middle Atlas plateau reservoir (northern Morocco).

From hydrogeological point of view, the Lias aquifer is a fractured reservoir of great importance, consisting in limestones and dolomites where the water flow occurs essentially along open fractures. His extension and continuation beneath the Saïs basin forms a confined aquifer very much solicited by agricultural pumping wells and the water supply of the two cities of Fes and Meknes (1.200 000 inhabitants both). The quantification and modelling of fracturing is thus of great interest for studying connectivity, transport properties and pollution vulnerability.

Figure 1 presents a map of northern Morocco and the location of the studied area (geological map of Morocco, sheet of El Hajeb).

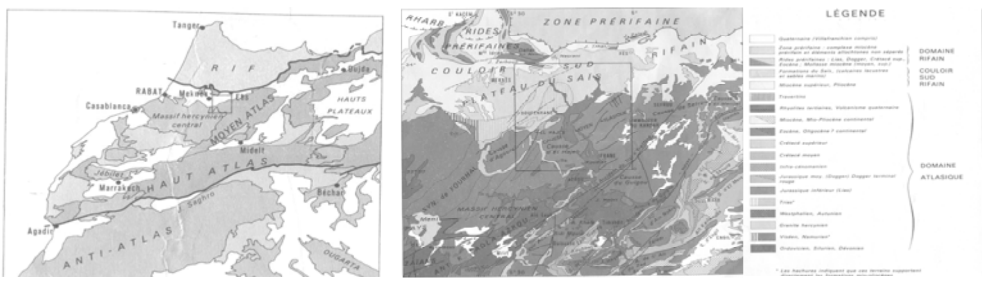

Figure 1: Middle Atlas location and geological setting

FRACTAL ANALYSIS

Fractals and capacity dimension

Fractures exist over a wide range of scales, from microfractures to largest faults. Their patterns display a self-similar geometry, at least at statistical sense, that repeat over various scales. This scaling behavior is described by a non-integer (or fractal) dimension varying in 2D from 1 to 2.

The box-counting method is generally used to measure the fractal dimension of the spatial distribution and scaling of fractures. A sequence of grids of different cell size ε is placed over the fracture map, the number of cells intersected or containing a fracture is counted. The fractal relation is:

$$N\varepsilon^{D} = 1$$

The fractal dimension is:

$$D = \frac{\log N}{\log(1/\varepsilon)}$$

where N is the number of cells containing fractures, ε is the length of the side of the cell, and the fractal dimension D is the slope of straight line segments fitted to the N, $1/\varepsilon$ plotted on logarithmic axes (Barton, 1995). D is the so called box-counting dimension or capacity dimension, generally noted D_0.

The application analysis to a fracture traces from Al Hajeb map (Fig.2) Figure 3 gives an estimated fractal dimension D_0= 1.74.

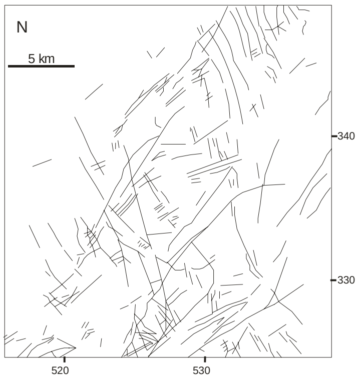

***Figure 2**:* Trace map of fracture network in El Hajeb region

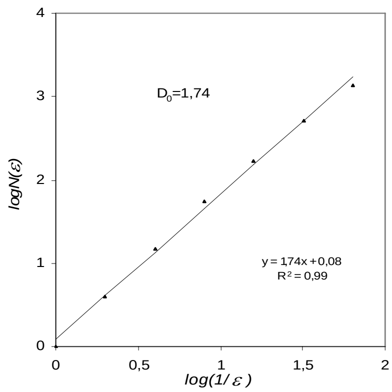

Figure 3: Box-counting fractal dimension estimation

MULTIFRACAL ANALYSIS

Theory

The major characteristic of fractal sets is their scaling properties related to self-similarity. For monofractals, scaling can be described by only one exponent (fractal dimension). This is not the case for natural fractal sets like fracture networks, which are multifractals. These objects can then be entirely described by a spectrum *D(q)* fractal exponents, the generalized dimensions, where the fractal dimension is D_0 and the function *D(q)* is the multifractal spectrum (Halsey *et al.*, 1986 ; Mandelbrot, 1989 ; Schertzer & Lovejoy, 1989 ; Everetz & Mandelbrot, 1992).

Application

From the map, we obtained the multifractal exponents *D(q)* by using the box-counting algorithm. The fracture map is covered by N_n boxes of size ε_n, where the subscript *n* indicates the *n*th generation scale. We associate the content of each box with measures F_i, which in our case a fracture intensity index defined as:

$$F_i = \frac{l_i}{L}$$

where l_i is the total fracture length in the *i*th box (i=1,.....,N_n), and *L* is the total length of fractures on the map.

Thus, the fracture intensity is the measure and its support is the natural area represented by the 2D map itself.

The generating function $\chi(q,\varepsilon)$, defines the multifractal spectrum in terms of the mass exponents $\tau(q)$, where $\tau(q)$ is a real parameter given by:

$$\chi(q) = \varepsilon_n^{-\tau(q)}$$

And can be calculated by using:

$$\chi(q) = \sum_{i=1}^{N_n} P_i^q$$

Finally we define:

$$D(q) = \frac{\tau(q)}{(q-1)}$$

The procedure of calculating the multifractal spectrum is carried out with *q* in the range -12 to 12 with an increment of 1.

Results

Figure 4 displays the mulifractal Spectrum of fracture traces of figure 2.

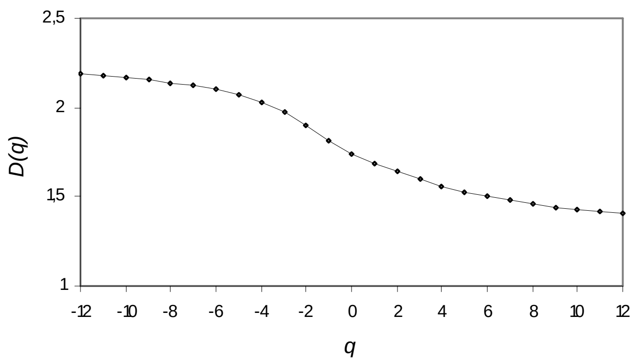

Figure 4: Multifractal spectrum. *D(q)*: dimension of order *q*.

D(q) exhibits a characteristic behavior, decreasing with increasing *q* (Hentshel & procaccia, 1983). The results suggest that the spatial distribution of fracture intensity has not a homogeneous fractal structure but a heterogeneous one, with generalized positive fractal dimensions D_0= 1.74>D_1=1.68>D_2=1.63>……….>D_{12}=1.40). D_0, D_1, and D_2 are respectively the capacity, information and correlation dimension. The value $D_{12}=D_\infty$ is the fractal dimension of the most intensive clustering in the heterogeneous set.

It obvious that one fractal dimension is not enough to describe the fracture properties related to scale. A full spectrum of generalized dimensions is then required to take into account the fracture clustering which affect connectivity.

POWER-LAW DISTRIBUTION OF FRACTURE LENGTHS

Several field studies have demonstrated that fracture populations have a power-law length distribution (Scholz & Cowie, 1990; Davy, 1993).

Natural fracture length distributions thus show to obey to this fractal law:

$$N(>l) \propto l^{-c}$$

N is the number of fractures having a length equal or greater than *l*, and *c* an exponent varying generally between 1 and 2.

The exponent *c* is linked to the amount of short and large fractures, and it has important consequence on connectivity properties (Bour & Davy, 1997).

Application

The cumulative frequency distribution of fracture length (n=976) shows a power-law behavior with exponent *c*~1.99 (Fig.5), which agree well with literature (Cladouhos & Marett, 1996).

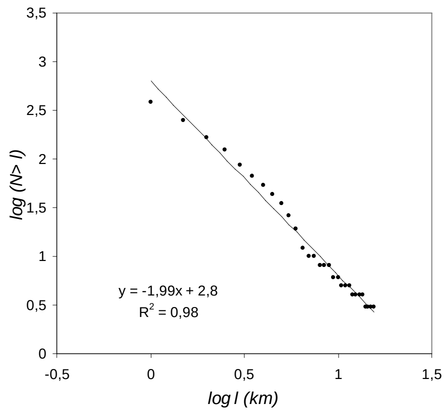

Figure 5: Fracture length distribution

Over more than two decades a best fit was obtained, with a good linearity between 1 and 10km length. The maximum extrapolated fracture length is about 25km.

This power-law can be used to estimate also the number of small fractures; the same law and the same fractal exponent will be valid for the entire Middle Atlas Lias reservoir.

Connectivity

According to numerical modeling (Bour & Davy, 1997) and on the basis of the percolation theory (Stauffer & Aharony, 1995), the connectivity of fractured media depends upon the power-law exponent and the fracture density. The small and large fractures will contribute to connectivity with an amount or ratio depending on exponent c. In our case, c~2 may be a critical value where 50% of fractures belong to the infinite cluster or correlation length. Berkowitz *et al.* (2000) analyzed the fracture connectivity on the light of relation of exponent c to capacity fractal dimension D_0, for $c>D_0$, the connectivity does not depend of scale, inversely for $c<D_0$, the connectivity threshold is reached only at a critical value. In the middle Atlas Lias fractured reservoir, with D_0=1.74 and c=1.99, it is attempting to say that fracture connectivity is largely independent on scale.

Another application to Sefrou region shows a similar behavior with the same power-law exponent and a well developed spectrum of singularities, some possible hydrogeological implications could be drawn.

CONCLUSION

Fracture networks in the Middle Atlas aquifer have multifractal structure characterized by the full singularities spectrum, and exhibiting its scaling behavior. The power-law fracture length distribution shows a scale-invariance.

The fractal exponent could be attributed to the entire reservoir, both at surface and in confined aquifer. The fracture connectivity seems to be independent of investigation scale.

The extrapolation to 3D analysis based on fractal mathematics and percolation theory will constitute an interesting field of research for a better understanding of fracture geometry and reservoir hydraulic properties. The management of data in a GIS could help to a better optimization of water prospecting of this region, focusing in fractured connected media to reduce dry wells.

BIBLIOGRAPHY

Barton C.C., Fractal analysis of scaling and spatial clustering of fractures. In C.C. Barton & P.R. LaPointe (eds.). Fractals in the Earth Sciences. Plenum Press: 141-178, 1995.

Berkowitz B., Bour O., Davy P., Odling G., Scaling of Fracture Connectivity in Geological Formations. Geophys. Res. Lett, 27, 14, 2061-2064, 2000.

Bour O., Davy, P. Connectivity of Random Fault Networks Following a Power-Law Fault Length Distribution. Water Ressour. Res., 33, 7, 1567-1583, 1997.

Cladouhos T.T., Marrett R., Are Fault Growth and Linkage Models Consistent with Power-Law Distribution of Fault Length? J. Struct. Geol., 17, 863-873, 1996.

Davy P., On the frequency-length distribution of the San Andreas fault system. J. Geophys. Res., 98, 12414-12151, 1993.

Evertsz C.G.C., Mandelbrot B.B. Multifractal Measures. In Peitgen H.O., Jurgens H. Saupe D. (eds). Chaos and Fractals. Springer Verlag: 849-881, 1992.

Halsey T.C., Jensen M.H. Kadanoff L.P., Procaccia I., Shraiman B.I., Fractal measures and their singularities: the characterization of strange sets. Phys. Rev., A 33, 1141-1151, 1989.

Geological Map of Morocco, 1975 Sheet of Al Hajeb (1: 100 000). Notes et Mém. Sér. Géol. Maroc, n°160.

Mandelbrot B.B., 1982 The Fractal Geometry of Nature. Freeman, NewYork, pp 468.

Schertzer D., Lovejoy S. Nonlinear variability in geophysics: multifractal analysis and simulation. In L. Pietronero (ed.). Fractals Physicals Origin and Properties. Plenum Press: 49-79, 1989.

Scholz C.H., Cowie P.A., Determination of total strain from faulting using slip measurements. Nature, 346, 873-879, 1990.

Stauffer D., Aharony A., 1992 Introduction to Percolation Theory. Taylor and Francis, Bristol, pp 181.

Turcotte D.L., 1997 Fractals and Chaos in Geology and Geophysics. 2d Ed., Cambridge, pp 398.