

Can Betweenness Centrality Explain Traffic Flow?

Aisan Kazerani and Stephan Winter

Department of Geomatics, The University of Melbourne, Victoria 3010, Australia
a.kazerani@pgrad.unimelb.edu.au, winter@unimelb.edu.au

INTRODUCTION

Centrality measures describe structural properties of nodes (and edges) in a network. Betweenness centrality (Freeman 1977) is one of them, characterizing on how many shortest paths a node is. So far, network analysis concentrates on structural, i.e., topological properties of networks, and on static formulations of centrality. Although travel networks can be studied this way, they deviate from other networks in two significant ways: their embeddedness in geographic space is relevant, and their dynamic properties can not be neglected. For example, a physical urban street network constrains travel behavior in a way that people seek to satisfy their demands from physically near, not topologically near resources. Also, a physical street can have significant temporal constraints, such as night time closures, dynamic lane allocation, or current traffic volume, besides of slow rates of change in the network itself. This means, it is not appropriate to compare traffic flow on street networks with traditional betweenness centrality.

Traffic flow is the process of physical agents moving along an urban travel network. These agents are autonomous, purposeful, flexible, and volatile. They establish a *social* network: agents near to each other can communicate and interact (other social ties, like kinship or friendship, are not considered here). Since the agents are mobile this social network is highly dynamic. Also agents are volatile. They enter traffic at any time, and leave as soon as they have reached their destination. The places where they emerge or disappear are distributed over space and time, but not in a random manner. Additionally, agents in urban traffic are purposeful. They have individual travel, sensing and communication capabilities, maybe even preferences, and a specific travel demand (to reach a destination by a specified time or specified costs). Especially, during travel they can interact with their fellow agents, be it by coordination (communication) or collaboration (transport), and they can sense and act in their physical environment, and thus, change their travel plans at any time to satisfy their travel demand. This means travel plans—if not the underlying travel demand itself—can be dynamic. This social network of agents in traffic can also be characterized by centrality measures; however, these measures are attributes of the agents, not of the nodes of the physical travel network, and they are constantly changing—hence, infeasible to track in a central database.

With all these behavioral observations of urban traffic at hand, we are interested in whether, and if, to what extent the physical structure of the network, characterized by betweenness centrality, determines the traffic generated by the agents. Betweenness centrality is based on shortest paths in the network, and the typical (implicit) assumption of rational agents suggests expecting that they travel shortest paths. Accordingly one can expect that nodes on many shortest paths in the urban travel network attract much traffic, and by this way, betweenness centrality can characterize the patterns of traffic flow or traffic density. This view is supported by evidence reported in the literature (see, e.g., Hillier et al. 1993; Penn et al. 1998; Jiang 2008b). But two factors are shaking our confidence in this argument. First, we know that human agents act at most bounded rational (Gigerenzer 2008), and hence, their chosen distance function in determining shortest paths is likely not to be topological (alone). Second, the depicted dynamics of travel behavior cannot be found out of the characteristics of a static network. Therefore, whether betweenness centrality can explain traffic flow is a valid question. We will try to prove the hypothesis that betweenness centrality of the physical travel network is insufficient to explain traffic flow.

In principle the hypothesis can be approached from two sides. From one side, we can try to adapt traditional betweenness centrality for travel networks to capture spatial embedding and dynamics in this measure. Alternatively, we can try to capture and study the behavior of betweenness centrality in the social network of the traveling agents. In this paper we take the first approach.

To investigate the hypothesis we first discuss the shortcomings of using betweenness centrality in the physical travel network for predicting traffic flow, and the necessity to consider travel behaviour for this task (Section 3). We then go on and suggest some amendments to the betweenness centrality measure. We demonstrate that the theoretically derived amendments to betweenness centrality yield significant differences in the centrality values, hence, substantiate our hypothesis.

LITERATURE REVIEW

Centrality is a fundamental concept in network analysis. While in the past the role and identity of central nodes were investigated, now the emphasis is more shifted to the distribution of centrality values through all nodes (Crucitti et al. 2006). Centrality, though under various terms, is stressing the idea that some places—nodes in a network—are more important than others (Wilson 2000).

Bavelas (1948) was the first to realize that central individuals in a social network very often play a prominent role in the group, or in other words a good location in the network structure corresponds to power in terms of independence, influence and control on the others. He applied the idea of centrality to human communication; he was interested in the characterization of the communication in small groups of people and assumed a relation between structural centrality and influence and/or power in group processes. Since then, various measures of structural centrality have been proposed over the years to quantify the importance of an individual in a social network (Wasserman and Faust 1994). This made centrality also a core concept for the analysis of social networks, and betweenness is one of the most prominent measures of centrality. It defines centrality in terms of the degree to which a point falls on the shortest path between others and therefore has a potential for control of communication (Freeman 1977). Therefore in order to investigate the relevance and relation of centrality in physical networks and travel characteristics, betweenness centrality can be a useful tool for this comparison.

Taking the individuals of a social network as nodes, and their links as edges, is following a *primal* representation (Porta et al. 2006b). In spatially embedded networks, such as street networks, punctual geographic entities—street intersections, settlements—are turned into nodes, and their linear connections—street segments, or streets—are turned into edges. It is possible to characterize and discuss urban street networks on this basis (Crucitti et al. 2006), by using the same framework of network topology that has been applied for studying social networks (Freeman 1979; Wasserman and Faust 1994).

While most urban analyses by GIS use street intersections and segments as the building block of the street pattern, urban planners introduced the concept of axial lines for urban space analysis (Hillier and Hanson 1984). The axial line is the longest line of visibility in a convex space—here a street scene, and hence representing a street or a part of it. *Space Syntax*, the related methodology of urban analysis, has been raising growing evidence of the correlation between the so-called integration of urban spaces, a closeness centrality in all respects, and phenomena as diverse as crime rates, pedestrian and vehicular flows, retail commerce vitality and human wayfinding capacity (Hillier 1996). In fact, after the seminal work of Hillier and Hanson a rather consistent application of the network approach to cities, neighborhoods, streets and even single buildings has been developed.

Axial lines are an example for a *dual* representation. A dual representation, or the representation of a dual conceptualization of a network, represents lines in the network as nodes, and a relationship

between lines by edges (note that a dual representation does not lead to a dual graph, i.e., the dual of the dual is generally not the original graph.). This way, axial lines form the nodes, and intersections of axial lines establish links between the nodes. But dual representations can also be built from street intersections and segments (Winter 2002), or whole streets as formed by a good continuation principle (Thomson and Richardson 1999) or by an attribute of street segments, the street name (Jiang and Claramunt 2004). The resulting graphs can be investigated for centrality in networks in the same manner (Porta et al. 2006a).

Whether computed from primal or dual representations, centrality measures are frequently suggested to characterize the flow of traffic on a street network (e.g., Crucitti et al. 2006; Porta et al. 2006a; Porta et al. 2006b; Jiang and Liu 2007; Jiang 2008b), and some report of correlations between betweenness centrality and traffic counts of 0.8 and higher (Hillier et al. 1993; Penn et al. 1998; Jiang 2008a). These correlations were found either for axial lines or for named streets. A correlation between traffic and the travel network is not surprising since the urban street network emerged around traveling needs, and grew with evolving needs.

As a complex system (Simon 1962), traffic (Bellomo et al. 2002) shows emerging patterns in its flow: a variable density—in the extreme case congestion (demand beyond capacity)—varying over time. Complex networks whose edges have been assigned a given weight (the flow or the intensity) can be generally described as *weighted graphs* (Harary 1969). With traffic flow being a dynamic process, the graph weights should be a function of time. Another reason to introduce weighted graphs is the spatial embeddedness of the network, which leads to weights based on travel costs such as distance, travel time, or fares. Also these travel costs can be a function of time. Naturally, traffic flow is a (dynamic) characteristic of street segments, not of axial lines or any conceptualization of streets.

The assumed correspondence between traffic and network structure was also behind the suggestion to link centrality with prominence, i.e., properties of social or individual cognitive salience, arguing that streets frequently travelled are better known (Tomko et al. 2008). Only few have related centrality measures to individual movement so far (e.g., Batty et al. 2003).

(DE-)CONSTRUCTING BETWEENNESS CENTRALITY

The classical method of calculating betweenness centrality was proposed by Freeman (1977). Betweenness centrality C_i^b is based on the idea that a node is more central when it is traversed by a larger number of the shortest paths connecting all pairs of nodes in the network. Let $G(V, E)$ denote a graph G consisting of vertices V pair-wise connected by edges E . Let v_i, v_j and v_k be distinct vertices of G . Let $|\sigma_{jk}|$ denote the number of shortest paths between v_j and v_k , and $|\sigma_{jk,i}|$ the number of such paths leading through v_i . Betweenness centrality of the vertex v_i is defined in $G(V, E)$ as follows:

$$C_i^b = \sum_{\substack{j,k \\ j \neq k \\ j,k \neq i}} \frac{|\sigma_{jk,i}|}{|\sigma_{jk}|}, \quad \text{or normalized:} \quad \bar{C}_i^b = \frac{C_i^b}{(|V|-1)(|V|-2)} \quad (1)$$

As such betweenness centrality is sensitive to the conceptualization of the physical travel network, whether it is by street segments, axial maps, a good continuation principle, or by its named entities (Figure 1). Furthermore, analyses can be applied to the primal or dual graph representations of these conceptualizations (Figure 2, see also Jiang and Liu 2007). Both aspects influence the observed

values of betweenness centrality. The graphs do not match, and thus, betweenness centrality measures of the nodes are different in each case.

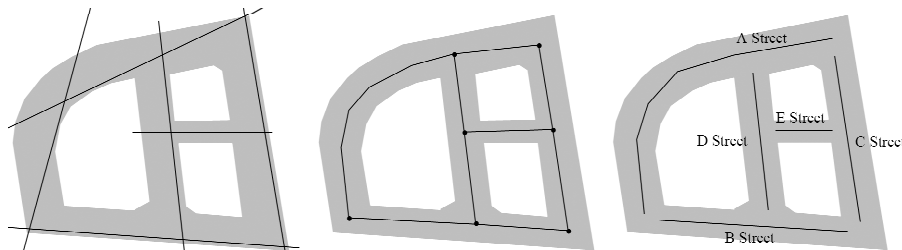


Figure 1: An urban street environment conceptualized as (a) axial map, (b) street centerline segments, and (c) named streets.

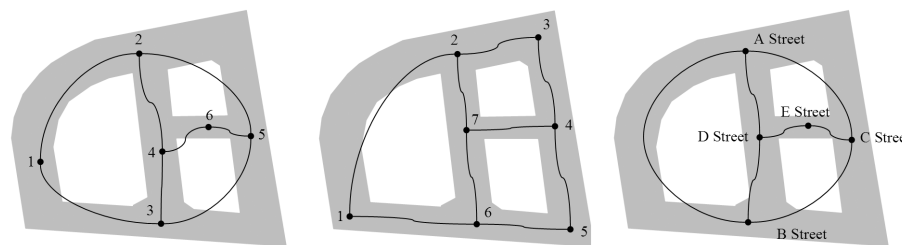


Figure 2: The three conceptualizations of Figure 1 represented by graphs: nodes represent (a) axial lines (dual), (b) street intersections (primal), and (c) named streets (dual).

Additionally, betweenness centrality is sensitive to the chosen distance function. For example, topological distance would weight each edge by 1, and geometric distance would weight each edge according to its length. The resulting shortest paths between two nodes differ, and accordingly, betweenness centrality would be computed from different sets of shortest paths. Virtually all analyses so far are based either on topological distance or on geometric distance. To our knowledge, other options are even not supported by any current network analysis software.

A related observation against the expectations set out in betweenness centrality is about human wayfinding behavior. People find their way with bounded rationality, i.e., based on incomplete individual network knowledge, inaccurate and systematically distorted weights of edges (Stevens and Coupe 1978), a hierarchic cognitive map (Hirtle and Jonides 1985), and heuristics instead of optimal algorithms. For the same reason—cognitive efficiency in an uncertain world—they show a habitual wayfinding behavior. This means that people do not (necessarily) follow topologically or geometrically shortest paths. They may have more complex cost functions in mind, but they also apply heuristics. Accordingly, observed traffic flow should deviate to some extent from the sum of all shortest routes between traffic sources and sinks. A strong support of this claim is Braess' paradoxon (1969), where he shows that a new shortcut in a network may lead—in a perfectly rational decision—to on average longer travel times, because everybody will try to use it and nobody is able to estimate the impact on capacity.

A final observation is about the emergence of traffic: Sources and sinks of traffic are irregularly distributed in space and change over the course of the day (see, for example, the daily commuting patterns) or other temporal cycles. A spatially irregular distribution of traffic demand contradicts the fundamental assumption of betweenness centrality that traffic happens between nodes, and equally likely from all nodes to all nodes.

So far, we have collected sufficient reason to argue that the hypothesis is correct and we need modified versions of betweenness centrality to characterize traffic flow. This means we do not necessarily need further experimental evidence. However, additional experimental evidence on actual traffic flow could come from several sources.

One source of evidence can be traffic counts, as collected nowadays automatically by traffic authorities. Traffic counts provide direct measures of betweenness centrality according to traffic flow for each (measured) street intersection. Since no links can be established to the traveling agents, the relation of this measure to traveled routes remains unclear. The consequence is that the results have no explanatory power, and hence, results cannot be transferred to another city.

Another source of evidence can be origin-destination matrices of daily movement patterns, as they are typically compiled from census data. For example, the Australian Bureau of Statistics maintains a *Journey to Work* dataset containing the origin, the method of transport, and the workplace destination of citizens. Origin-destination matrices can, for example, feed traffic simulations (Nagel et al. 1998; Balmer et al. 2004). Simulation, again, depends on the detail of knowledge put into the system; here especially the expectations by which criteria people will choose their travel routes from start to destination. Simulated traffic counts characterize betweenness centrality in the same manner as actual traffic counts, but since simulation can be applied to any network, the observation of betweenness centrality is cheaper and more expressive this way. Note how both measures of betweenness centrality have become time dependent.

Thus what remains to do is studying possibilities to modify the formula of betweenness centrality (Equation 1) such that it reflects better traffic flow. Such a modified formula should facilitate a theoretical prediction of traffic flow, without going back to actual or simulated traffic counts. As such it will have strong explanatory power.

DEVELOPING A PRELIMINARY MODEL

Developing a modified betweenness centrality measure finally has to address any of the above identified issues. In this paper we focus on one issue only, developing a preliminary model and demonstrating by this way the principle feasibility of this goal. The other issues are left for future work. The selected issue—betweenness centrality reflecting the locations of origins and destinations of recorded trips—will be compared to classical betweenness centrality—the one based on the topological structure of the street network.

An environment that is large enough to discuss the details of the model and small enough to be understood completely is shown in Figure 3. On the left side a primal graph of a street centerline conceptualization is shown. The graph consists of five vertices $V = \{v_1, v_2, v_3, v_4, v_5\}$ and five edges $E = \{e_1, e_2, e_3, e_4, e_5\}$. Since the graph is embedded, edges can be characterized by their geometric length, or by any other travel cost function. These costs are shown in Figure 3 as well. On the right side of Figure 3 the dual graph G' is added in dashed lines. Edges E of the primal graph become vertices V' , and adjacent primal edges e_i and e_j become linked by edges e'_{ij} in the dual graph. Also these edges e'_{ij} can be weighted, although the semantics of weights are different now. For the example let us choose travel costs from the center of e_i to the center of e_j . For example,

$$w(e'_{13}) = \frac{w(e_1) + w(e_3)}{2}.$$

In this scenario, betweenness centrality is computed for (primal) edges e_i , because road traffic is originating from and ending along edges, not vertices (note that other travel networks, such as train networks, behave differently). For the results in Table 1, Equation 1 is applied for uniform

(topological) distance as well as for weighted (geometric) distance. For example, e_1 is on the topologically shortest paths from e_2 to e_5 and vice versa, and on one out of two topologically shortest paths from e_3 to e_5 and vice versa. In terms of weighted shortest paths, e_1 is on the shortest path from e_2 to e_5 and vice versa, and on the (single) shortest path from e_3 to e_5 and vice versa.

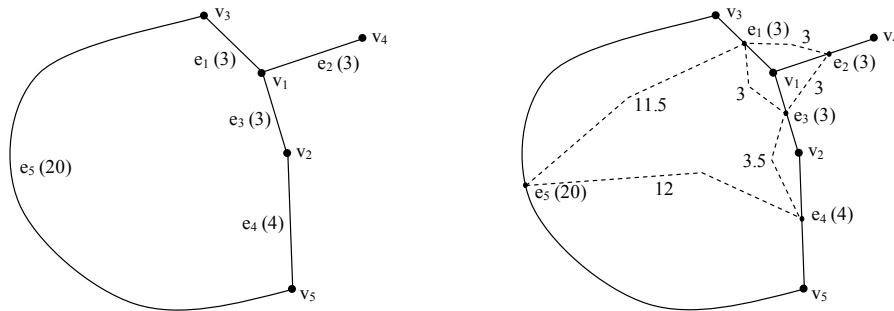


Figure 3: Another street centerline graph in primal (left) and dual representation (right, dashed).

For one, this (traditional) betweenness centrality is not dynamic, although temporal dependency can easily be added by choosing weights as a function of time. But also it is still unsatisfactory in terms of considering travel from every edge to every edge happening with the same likelihood, and by using an unreliable geometric distance function. In the next step these issues will be resolved.

C_i^b	dual, topological distance	dual, geometric distance
e_1	3	4
e_2	0	0
e_3	3	4
e_4	1	0
e_5	1	0

Table 1: Betweenness centrality of the edges $e_i (=v'_i)$ in Figure 3, right.

Individual travels originate at particular places along street centerlines, and end at particular places. Furthermore, these travels are happening in space-time; they have a start and end time. Traffic is then the summary of these individual travels, and traffic flow has a behavior over time. Apparently there is more travel demand in peak hours, when people travel to work or come back home from work. On weekday mornings the places where people live are the sources of traffic, and the workplaces are the sinks. This pattern reverses for their commuting trip back in the afternoon. While commuting is the major factor in generating urban traffic, other travel demands are superimposing the resulting traffic patterns, travel demands such as shopping or leisure.

Let us assume that we know the travel demand of a population, specified in time. Figure 4 shows the street centerline graph of Figure 3 enriched with all places $P = \{p_1, \dots, p_{10}\}$ that are relevant for a particular time window, let us say, 9am to 10am on a weekday. For convenience the figure also shows the edge weights split by the locations of the places. In this graph true weighted shortest paths can be computed.

The individual travel demand between the places is listed in Table 2: from place p_1 is a demand to travel to p_8 and to p_7 , and so on. Assuming that the individual travelers choose the geometric shortest paths to their destinations, Table 2 also lists the vertices of the street centerline graph that are passed. In total, nine shortest paths can be found. Now a modified betweenness centrality measure \tilde{C}_i^b for

primal vertices v_i can be computed by counting all shortest paths through v_i divided by all shortest paths possible: This measure is:

$$\tilde{C}_i^b(\Delta t) = \frac{|\sigma_{pp,i}|}{|\sigma_{pp}|} \quad (2)$$

Results for the example are shown in Table 3 (center column), and for comparison also the traditional betweenness centrality for the vertices are reported (right column).

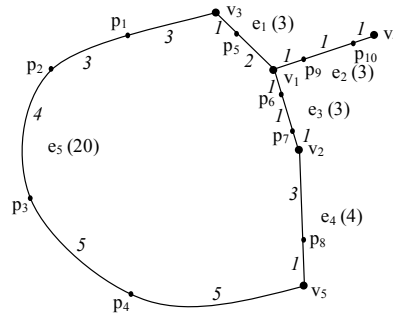


Figure 3: The street centerline graph of Figure 3 enriched with places and split edge weights.

Place	travel demand	shortest paths via
p_1	p_8	v_3, v_1, v_2
	p_7	v_3, v_1
p_2	p_8	v_3, v_1, v_2 or v_5
	p_9	v_3, v_1
p_3	-	
p_4	p_8	v_5
p_5	-	
p_6	-	
p_7	-	
p_8	-	
p_9	-	
p_{10}	p_9	-
	p_6	v_1
	p_5	v_1

Table 2: Individual travel demand between 9am and 10am on a weekday.

	\tilde{C}_i^b	C_i^b
v_1	6/9	6/12
v_2	2/9	3/12
v_3	4/9	2/12
v_4	0/9	0/12
v_5	2/9	1/12

Table 3: Modified betweenness centrality of primal vertices in the studied time interval Δt .

Also alternative betweenness measures that are not based on shortest paths have been proposed (Freeman et al. 1991; Newman 2005), which may have potential for bringing in other aspects of traffic patterns.

CONCLUSIONS AND FUTURE WORK

This paper studied betweenness centrality for its potential to characterize traffic flow in street networks. Special focus was on the characteristics of the physical street network and its various conceptualizations and representations as a reliable tool for predicting traffic flow. It turns out that what was designed to characterize the design or outlay of physical networks is not suited for the dynamic processes going on in these networks. A number of issues have been identified that need to be addressed in a modified version of betweenness centrality that applies to the dynamic processes.

For a single issue, origin-destination matrices, or the non-uniform distribution of travel demand, this modification was developed and successfully demonstrated. The measures deviate from traditional betweenness centrality, and they become time dependent. They predict actual traffic counts over given time intervals.

This paper suggests many questions and directions for future work:

- In our discussion we have compared the characteristics of the physical (street) and the social (travelers') network. While the presented modified measure relies on physical properties of places of origins and destinations, the highly dynamic social network should have similar properties, since it is conceptually closely related to actual traffic counts.
- The correlation of (traditional) betweenness centrality and traffic reported by others relied on conceptualizations such as axial lines or named streets, while we have used even finer spatial granularity than street centerlines. We argue here that correlations by axial lines or named streets must be coarse generalizations since they are not time dependent and do not consider the non-uniform distribution of travel demand. However, future work can extend our model towards spatially or temporally aggregated data of coarser granularity.
- Further work can verify results by comparison with actual traffic counts. This will be especially helpful to validate the current assumption that people choose the geometric shortest path, which is not necessarily the case as some literature in spatial cognition suggests. For example, hierarchies within the street network are not yet considered in this work, but can be considered in future research.

ACKNOWLEDGEMENTS

This work is supported under the Australian Research Council's Discovery Projects funding scheme (project number 0878119). The comments of anonymous reviewers have helped to improve this paper.

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